Granular Mechanics of Geomaterials
(presentation in Cassino, Italy, 2006)

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Where am I from?

[Map of Japan highlighting Tsukuba, Kyoto, and Osaka]

[Pictures of buildings and a campus]

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What is granular mechanics?

**Particle characteristics**
(size, shape, crushability etc.)

**Particle interaction**

**Direct simulation**
(by DEM etc.)

**Constitutive model as solid or fluid**

**Simulation as continua**
(by FEM, SPH etc.)

No element test!?
Lunar exploration

Mechanical properties of Lunar soil is needed.

But only TWO TC test results is available.

Lunar regolith:
very angular well graded sand

Lunar sourcebook, 1991
Why granular mechanics?

*TRUE prediction from particle information

eg.
Mechanics of crushable soil
Mechanics of unsaturated soil
Mechanics of cemented soil
and more…

*UNIQUE constitutive model
more convincing, more rational
Contents:

1. Theory of granular mechanics
   Chang and Misra 1990, Matsushima and Chang 2006

2. Micro X-ray CT experiment
   Matsushima et al. 2002-

3. Image-based DEM simulation
   Matsushima 2002-

   incl. recent application to Lunar exploration
   Matsushima 2006
Basic theories

Molecular dynamics

Statistical mechanics for molecules
(sparse, non-frictional)

Mechanics of solid frictional particles
Kinetic theory
(sparse \rightarrow behaves as a fluid)

+ 

Mechanics of geomaterials
(dense \rightarrow behaves as a solid)
Uniform strain model (Chang and Misra 1990)

stress $\sigma$ \quad strain $\varepsilon$

contact \quad contact

force, $F$ \quad displ. $\delta$

*Grain centroids stick to continuum deformation field
*Grain rotation coincides with continuum rotation
$\rightarrow$ contact displ. $\delta$
Global $\rightarrow$ local coordinates

$$\delta^L = R^T \cdot \delta$$

Force-displ. relation at contact

$$\begin{pmatrix} f_n \\ f_s \end{pmatrix} = \begin{pmatrix} k_n & 0 \\ 0 & k_s \end{pmatrix} \cdot \delta^L$$

Global local coordinates

$$f = R \cdot f^L$$

Stress

$$\sigma^T_L = \frac{1}{V_R} \sum_c (l^t \otimes f^t)$$

Sum of contacts in $V_R$

work at contact = work as continuum
Elastic solution

Assuming equal-sized isotropic sphere packing...

$$E = \frac{2r^2N}{15V} (2k_n + 3k_s) \left( \frac{5k_n}{4k_n + k_s} \right)$$

$$\bar{v} = \frac{k_n - k_s}{4k_n + k_s}$$

$$\frac{N}{V} = \frac{3n}{4\pi r^3 (1 + e)}$$

Overall Young’s modulus and Poisson ratio can be described by contact stiffness

**e-n relation is necessary**

$n$: coordination number
Applying Hertz-Mindlin contact law... (Johnson, Contact mechanics)

\[ \dot{f}_n = \left( \frac{\sqrt{3rG}}{1 - \nu} \right)^{2/3} \]

\[ f_n^{1/3} \dot{\delta}_n = k_n \dot{\delta}_n \]

\[ \dot{f}_s = \frac{2(1 - \nu)}{2 - \nu} k_n \left( 1 - \frac{f_s}{f_n \tan \phi} \right)^{1/3} \]

\[ \dot{\delta}_s = k_s \dot{\delta}_s \]

\[ \bar{E} = \frac{2r^2(5 - 4\nu)}{3(5 - 3\nu)} (9\sigma_0)^{1/3} \left[ \frac{GN}{(1 - \nu)V} \right]^{2/3} \]

\[ \bar{V} = \frac{\nu}{2(5 - 3\nu)} \]

\[ \bar{G} = \frac{\bar{E}}{2(1 + \bar{V})} = A \left( \frac{n(e)}{1 + e} \right)^{2/3} (\sigma_0)^{1/3} \]

\[ n(e) = 2.63 - 1.79e \]

\[ A = \frac{2r^2(5 - 4\nu)}{15(2 - \nu)} (9)^{1/3} \left[ \frac{3G}{4\pi r^3 (1 - \nu)} \right]^{2/3} \]

\[ \bar{G} \text{ is a function of confining pressure, } \sigma_0 \text{ and void ratio, } e \]

Smith et al. (1929) by the combination of closest and loosest packing.
Elastic solution (continued..)

Cf. Hardin & Richart

\[ \overline{G} = B \cdot \frac{(2.17 - e)^2}{1 + e} \left(\sigma_0\right)^{1/2} \]
Elastic solution (continued..)

\[
\sigma_e G = \frac{(2.17 - e)^2}{1 + e} (\sigma_0)^{1/2}
\]

Cf. Hardin & Richart

\[
G = B \cdot \frac{(2.17 - e)^2}{1 + e} (\sigma_0)^{1/2}
\]

Chang (A=280)  
H-R (B=840)  
experiment (Iwasaki et al.)
Elastic solution (continued..)

Note:

$A=280$ corresponds to $G=73$ (GPa), $\nu=0.25$ (sand particle as a solid)

Estimation of $\bar{\nu}$ is not good.
(Further research is necessary.)
Nonlinear model

● Loss of contact
  is considered by tension-free model

● Contact slip (plasticity)

\[ |f_s| \begin{cases} \leq -\mu f_n & \rightarrow \text{(elastic)} \\ > -\mu f_n & \rightarrow \text{(sliding)} \rightarrow |f_s| = -\mu f_n \text{sign}(f_s) \end{cases} \]

Analytical solution is not available.

\[ \rightarrow \text{A set of branch vectors are assumed and the solution is obtained numerically.} \]
Basic response (biaxial compression)

Material does NOT yield!

stress-strain curve    dilation curve
Basic response (biaxial compression)

Why?
Contact of loading direction does NOT slip.
DEM study

periodic boundaries at both directions

No grain rotation

grain rotation free

stress ratio $\sigma_1/\sigma_2$

maximum shear strain $\gamma$

volumetric strain $\varepsilon_v$

maximum shear strain $\gamma$
DEM study

Force chain in granular assembly

Contact normal force distribution
Buckling of granular column

(Matsushima and Chang, 2006)

- Assign minimum aspect ratio for contact force distribution
- Determine average contact force such that the stored energy in contact keeps constant

insufficient confining pressure ↓ buckling
Response of Buckling model

(Matsushima and Chang, 2006)

Buckling resistance controls the yield strength related to particle properties
Possibility of granular mechanics C.E.

Rational incorporation of particle properties
(size, shape, stiffness, crushability, contact cement, etc.)

Validation not only with macro response
but also with particle-level information

Detailed comparison with
Particle visualization experiment
More realistic DEM
is needed.
Particle visualization by Micro X-ray CT

At University Joseph Fourier, Grenoble, 2002
Objective

to obtain 3-D micro properties
(grain properties, microstructure
and its change due to external loading)
of some standard sands
with micro X-ray CT in SPring-8

Typical standard sands:

Toyoura sand: $D_{50} = 0.167$ (mm)
Ottawa sand:  $D_{50} = 0.174$ (mm)
Hostun sand:  $D_{50} = 0.408$ (mm)
S.L.B. sand:  $D_{50} = 0.681$ (mm)

(high resolution system is necessary)
Micro X-ray CT at Spring-8

The world's largest third-generation synchrotron radiation facility

Synchrotron (Electron is accelerated up to 8 GeV)
Storage ring facility

Experimental room

47 beamlines (BL)

X-ray CT is available at BL20B2 and BL47XU.
Outline of Spring-8

X-ray comes from here

CCD camera

Stage

Detector
X-ray CT system at SPring-8

Resolution:
BL20B2: 13 µm
BL47XU: 1.5 µm
Toyoura sand  dense
Example of CT image (BL20B2)

Toyoura sand

dense  medium dense  loose
Example of CT image (BL20B2)

Glass beads  Hostun sand  Ottawa sand
Example of CT image (BL47XU)

- Ottawa sand
- Toyoura sand
- Wakasa sand

Roundish

very angular

Lunar soil simulant (FJS-1)
Example of CT image (BL47XU)

Masado (crushable sand)  unsaturated condition
Micro Triaxial test (BL20B2)(1)

specimen diameter: 4.3mm
height: 10mm

pressure tank
loading motor
specimen
pressure cell
membrane
specimen diameter: 4.3mm
height: 10mm
Micro Triaxial test (BL20B2)(2)

CT scan of initial condition
↓
loading
↓
stop loading and CT scan (1.5 hours)

stress-strain curve

Toyoura sand
confining pressure=100 (kPa)
- dense ($e_{ini} = 0.719$)
- loose ($e_{ini} = 0.984$)
Micro Triaxial test (BL20B2)(3)

Toyoura (medium dense)

void increase within a shear band
Micro Triaxial test (BL20B2)(4)

Masado (cruchable sand)
0.351-0.64mm
(medium dense)

particle crushing is NOT predominant
Micro 1-D compression test

particle crushing after yield stress
Processing of CT image

Original image  After binarization  After pore-filling  After 1st erosion

After 2nd erosion  After 3rd erosion  Cluster labeling  Attribution
Adequate erosion cycles

- Toyoura sand, dense 030525a
- Glass beads, dense 030525d

Number of resulting clusters vs. frequency of equivalent diameter of cluster (μm) after 3rd erosion:

- Toyoura sand, dense 030525a
- Glass beads, dense 030525d

Number of resulting clusters vs. frequency of equivalent diameter of cluster (μm) after 5th erosion:

- Toyoura sand, dense 030525a
Grain identification

3-D image after attribution process
Grain identification

Identified Toyoura sand grains
Grain size distribution

percent passing by weight

Equivalent diameter (µm)

Glass beads
loose

Glass beads
dense

Toyoura sand
dense
Contact area and their normals

→ coordination number, fabric tensor, etc.
Coordination number (1)

- an030525a-3
  Toyoura sand dense \( (n=0.409) \)
  - Frequency (%): 8.51

- Smith et al. (1929)
  Lead balls loose \( (n=0.447) \)
  - Frequency (%): 6.89

- at030525e-3
  Glass beads loose \( (n=0.451) \)
  - Frequency (%): 7.18

Like a normal distribution
Coordination number (2)

$N_C \, n = \frac{e}{e + 1}$
Next step

*Detection of each grain motion
(translation and rotation)

*Detection of grain crushing
Image-based DEM
Background

* Rapid increase of computer abilities
  → **DEM** simulations with large number of grains

* For **quantitative** discussion…
  → **Precise modeling** of grains
     (contact model, grain shape, crushability, etc.) is necessary.

This study deals with

**GRAIN SHAPE MODELING**
Image-based modeling  (Matsushima and Saomoto 2002)

Irregular grain shape is described by a rigid connection of elements (circles or spheres)

To find an optimum positions and radii of the elements…

time marching computation (Dynamic optimization)

Each element moves and expands (or shrink) to get a better fitting to the target grain shape
Dynamic Optimization algorithm

Each surface point of a target grain gives an attraction to the closest element which directs from the centroid of the element to the surface point and its magnitude is proportional to the distance.
2D example (1)

Grain density   2.64 (g/cm²)
Spring constant
(normal)       1.0e9 (g/s²)
(tangential)   2.5e8 (g/s²)
Damping coefficient
(normal)      2.0e2 (g/s)
(tangential)  1.0e2 (g/s)
Friction coefficient between grains  27 (deg.)
Time increment  2.5e-8 (sec.)
2D example (2)

- 200 grains (Each grain is modeled with 10 elements)
- **Periodic boundary** at both sides
- Constant confining pressure (10kN/m),
- Lateral displacement is imposed at the bottom grains
2D example (3)

Dense specimen ($e_{ini}=0.20$)

Toyoura > Ottawa > Circles for peak strength
2D example (4)

Peak strengths with different initial void ratio

The modeling is verified qualitatively.
2D example (5)

Granular column structure during shear

Two grains are in contact with plural points
→Moment transmission →Higher strength
3D example: Toyoura sand (1)

X-ray CT result  
3D modeling

Toyoura sand model
3D example: Toyoura sand (2)

Grain size distribution:
DEM model = original CT data

Simple shear simulation
3D example: Toyoura sand (3)

Quantitative agreement with experiment if inter-particle friction angle=20(deg.)

Stress-strain curve
dilation curve

Matsushima 2004
3D example: Toyoura sand (4)

Toyoura sand DEM
\( \phi = 27 \text{ (deg.)} \)

Toyoura sand DEM
\( \phi = 20 \text{ (deg.)} \)

DEM (spheres)
\( \phi = 27 \text{ (deg.)} \)

TSS test (Toyoura sand)
(\( \sigma_n = 100 \text{ (kPa)} \))
(Pradhan et al. 1988)

initial void ratio \( e_{ini} \)

Peak strength for various void ratio
600 grains (10-spheres model) are fell down into a vessel

Periodic boundary in y-direction

Bottom grains are fixed into the bottom plate

Remove the right side wall
Lunar soil simulant(3): Parametric study

Effect of grain hardness

Effect of interparticle friction
Lunar soil simulant(4): model accuracy

2-elements model  3-elements model  4-elements model  7-elements model
Lunar soil simulant(5): model accuracy

![Graph showing the angle of repose vs. number of spheres for exp (FJS-1) and exp (glass beads).]
Particle shape effect: The mechanism

Contact force chain

Matsushima, P&G2005

Plural contact points between two grains
→ rolling resistance
→ overall shape is important
Conclusions

Application of Granular Mechanics into geomaterials has been tried for decades.

Some breakthrough has been found recently (in my opinion).

Recent progress of various technologies makes it push forward.
References

References


