Effect of grade changing due to grain crushing on the compressibility of granular materials

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Synopsis:
A series of analyses using a two-dimensional Discrete Element Method (DEM) with moment springs was conducted to investigate the effect of the grading change due to crushing on the compressibility of granular materials. The analyses simulated the one-dimensional compression tests on four samples simulating breakable particle assemblies with different gradations, and the relationship between the void ratio, $e$, and the grading state index, $I_G$, were studied. In the range of the normalized stress, $p/\sigma_0$, from 0.3 to 0.7, where grain crushing was prominently observed, $e$ decreased while $I_G$ increased intensively. Under high stress conditions ($p/\sigma_0 > 2$), $e$ of all four samples took similar value and the values of $I_G$ converged in a narrow range compared with the initial states, which implies the existence of the critical grading under the high stress conditions.

Keywords: grain crushing, grading, compressibility, Discrete Element Method (DEM).

1. Introduction

A considerable number of experimental studies have indicated that the volume change of granular materials is mainly caused by the particle rearrangement at lower stress levels, but by the particle crushing and the following rearrangement at higher stress levels (e.g. 1,2). Therefore the effects of the grading change is inevitable to understand the compressibility of the granular materials, especially in higher stress levels (e.g. 1,3).

The Discrete Element Method (DEM) (4) offers us a micromechanical insight into the grade changing process of granular materials. The following two methods are commonly used in DEM to simulate the grain crushing: 1) forming agglomerates of bonded elements (e.g. 5), and 2) replacing stressed particle by a group of smaller elements (e.g. 6). In this study the former method was adopted because it can directly simulate the abrasion and the changing shape of fragments. As Lim et al. (7) pointed out, however, this method allows the rolling of a particle with respect to another one without slipping in the situation where particles align in a straight or curved line, because a contact bond provides no resistance to rolling. In order to improve this insufficiency, the moment spring system based on the mechanical contact model developed by Jiang et al. (8) was introduced to the standard DEM; a series of analyses, simulating the one-dimensional compression tests of breakable granular materials, were carried out using two-dimensional assemblies of particles with different degree of initial composition.

2. Moment spring and bonding force induced Discrete Element Method

To avoid using additional mechanical parameters (9), the bonding force between two elements was assigned as follows:
1) the elements were initially positioned to overlap each other to some extent;
2) the initial repulsive force was calculated at the first step of the simulation;
3) the bonding force equivalent to the repulsive force was induced to cancel each other;

The moment spring employed the physically based mechanical contact model developed by Jiang et al. (8), which consisted of a set of parallel spring-dashpot-divider systems placed between two particles having contact width $B$ (Fig. 1 (a)), and the moment were calculated as follows;

$$M = K_m \theta_r = \frac{k_m B^2 \theta_r}{12},$$

where $\theta_r$ is the relative rotation, and $K_m$ and $k_m$ are the rotational spring stiffness at the contact and the stiffness of each spring-dashpot-divider.

The behaviour of contact model was classified into three states depending on $\theta_r$, i.e. plastic, softening and breakage states:
1) In the plastic state, where \( \theta_r \) is smaller than \( \theta_r^0 \) (Fig. 1 (a)), \( B \) is equal to the initial width, \( B_0 \), and the moment increases proportionally with increasing \( \theta_r \), where \( \theta_r^0 \) is a relative rotation at the bending fracture and given as \( \theta_r^0 = 2F_n / k_B B \), where \( F_n \) is the bonding force, according to the fracture mechanics.

2) In the softening state, where \( \theta_r^0 < \theta_r \leq 2\theta_r^0 \), \( B \) decreases with increasing \( \theta_r \) as \( B = B_0 \left( 2\theta_r^0 - \theta_r \right) \) due to the separation of some dividers (Fig. 1 (b)).

3) When \( \theta_r \) is larger than \( 2\theta_r^0 \), all dividers separate, which represents the breakage of two elements. Figure 1 (c) illustrates the induced moment curve with respect to \( \theta_r \).

**Figure 1.** Sketch of the mechanical contact model (9) (a) when \( \theta_r \) is less than or equal to \( \theta_r^0 \), (b) when \( \theta_r \) is larger than \( \theta_r^0 \) and less than or equal to \( 2\theta_r^0 \), and (c) moment curve respect to \( \theta_r \).

### 3. Single particle crushing simulation

To obtain the standard crushing strength of the particle, \( \sigma_0 \), a disk-shaped particle composed of 76 elements forming Hexagonal Closest Packing (HCP) was designed (Fig. 2). The particle was placed between two horizontal rigid plates, then the upper plate compress the particle under a constant speed. Considering the anisotropy of the particle composed of HCP elements, the initial inclination angle of the particle, \( \theta \), was set at each 5 degrees from 0 to 90 degrees and the peak strength was measured in every 19 cases. Figure 2 (a) and (b) show, as an example, the configuration of a particle with \( \theta = 20 \) degrees at the initial state and after breakage, respectively. The standard crushing strength was calculated by the Weibull’s equation (10) expressed as follows,

\[
P_s(d) = \exp \left( - \left( \frac{d}{d_0} \right)^m \left( \frac{\sigma}{\sigma_0} \right)^m \right),
\]

where \( P_s(d) \) is the unbreakable probability for a particle whose diameter and crushing strength are \( d \) and \( \sigma \), respectively; \( d_0 \) and \( \sigma_0 \) are standard diameter and crushing strength and \( m \) is the Weibull coefficient. When \( d = d_0, \sigma = \sigma_0 \) and \( m = 1, P_s(d) \) becomes 0.37, in other words, 37 % of particles remain unbreakable under \( \sigma_0 \) stress. Consequently, \( \sigma_0 \) was determined to be 1.35 (kN/m) by which the mean stress is normalized in the following chapter.

**Figure 2.** Single particle crushing simulation (a) at initial state and (b) after break.
4. Compression simulation on granular system

4.1 Simulation procedure

A series of one-dimensional compression simulations were conducted on four types of breakable granular assemblies and four types of rigid granular assemblies listed in Table 1; Fig. 3 shows grain size distribution curves and the initial configurations of these assemblies. Each assembly had a grain size distribution of log-normal type, with the same mean diameter ($D_{50} = 10$ mm) but a different value of standard deviation, $s$, defined by the following formula

$$f(D) = \frac{1}{\sqrt{2\pi s^2} D} \exp \left( -\frac{(\log_{10} D - \log_{10} D_{50})^2}{2s^2} \right),$$  \hspace{1cm} (3)

where $D$ and $f(D)$ are the diameter and the frequency of $D$, respectively. In the preparation of granular assemblies, circular particles of various sizes were generated to fit the designed gradation curves, and then these particles were replaced by the aggregates of mono sized elements whose diameters are 1mm. As shown in Table 1, the numbers of elements required were 4,000 to 5,000.

Table 1. Granular assemblies prepared for simulation.

<table>
<thead>
<tr>
<th>Case</th>
<th>Standard deviation of particle size, $s$ (mm)</th>
<th>Uniformity coefficient, $U_C$</th>
<th>Number of aggregates</th>
<th>Total number of elements</th>
<th>Breakage function</th>
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<tr>
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<td>0.000</td>
<td>1.00</td>
<td>55</td>
<td>4180</td>
<td></td>
</tr>
<tr>
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<td>1.11</td>
<td>56</td>
<td>4113</td>
<td>positive</td>
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<td>74</td>
<td>4973</td>
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<tr>
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<td>2.22</td>
<td>107</td>
<td>4645</td>
<td></td>
</tr>
<tr>
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<td>0.000</td>
<td>1.00</td>
<td>55</td>
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<tr>
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</table>

Figure 3. Grain size distribution curves of granular assemblies for simulation (left), and the initial particle configurations (right).

4.2 Simulation results

Figure 4 (a) to (c) compares the change in particle configuration of B023, a breakable granular assembly, with increasing stress level, $p/\sigma_0$, where $p$ is the mean stress; a considerable number of particles broke even at low stress level as shown in Fig. 4(b), which affected the particle size distribution as well as the compressibility. On the other hand, as indicated in Fig. 4(d), only particle rearrangement took place in R023, a rigid granular assembly, even at high stress level.
To analyze the particle size distribution quantitatively, a grading state index, $l_0$, was introduced according to the study by Wood and Maeda (11). Figure 5(a) shows the current grading curve and the fractal grading curve from which $l_0$ is defined as the ratio of area ABO to area ACO at fractal dimension of 2.5 (12). Figure 5 (b) compares the changing grade of B023 under various $p/\sigma_0$ and the fractal grading curve. As indicated in this figure, the grading is almost unchanged until $p/\sigma_0$ increases to 0.312; the fraction of smaller particle remarkably increases when $p/\sigma_0$ exceeds 0.416. The grading curves whose $p/\sigma_0$ are over 0.741 indicate that the assemblies consist of a large number of small particles whose size is equal to that of basic element, because the basic element is the minimum unit and uncrushable in this simulation. Figure 5 (c) compares the compression curves of all the simulation cases. This figure also shows that the void ratio, $e$, of the rigid granular materials are almost unchanged as previously indicated in Fig. 4 (d). On the other hand the void ratio of the breakable materials sharply decrease in the normalized stress range $p/\sigma_0$ from 0.3 to 0.7, then converge into a single point whose $e$ and $p/\sigma_0$ are 0.24 and 2.3, respectively. As shown in Fig. 5 (d) which illustrates the relationship between $e$ and $l_0$ for the breakable cases, the void ratio of B000 is smaller than that of B007 under low pressure ($p/\sigma_0 = 0.0234 \sim 0.416$). This is because the granular assemblies consisting of mono-sized particles like B000 can easily form the Hexagonal Clesest Packing (HCP), and accordingly the void ratio becomes smaller than that of B007 whose size dispersion prevents the particles from forming HCP (13). Except for B000 the void ratio generally decreased with increasing particle size dispersion or $l_0$. Under the high pressure ($p/\sigma_0 = 2.34$), $l_0$ takes considerably large values and the difference in $l_0$ among the 4 cases becomes small. These results imply that at considerably high pressure level, the grading of the granular materials converged into a critical state due to the grain breakage irrelevant to the initial composition. Figure 6 shows the relationship among, $e$, $p/\sigma_0$ and $l_0$ in a three-dimensional space similiary to that suggested by Wood and Maeda (11). This graph intuitively appeal the importance of understanding the characteristics of compression curves ($e$ – $\log p$ curve) with respect to the grading change.

5. Conclusions

A series of one-dimensional compression simulations was conducted on 4 types of granular materials composed of breakable particles having different particle size distributions, and the following conclusions were obtained.

1) Grain crushing was prominently observed in the range of the normalized stress, $p/\sigma_0$, from 0.3 to 0.7, and the void ratio sharply decreased (Fig. 5 (c)), meanwhile the grading state index drastically increased (Fig. 5 (d)).

2) Under higher stresses ($p/\sigma_0 > 2$), the void ratio of 4 simulation cases took almost similar values, and the range of grading state narrowed compared with the initial grading states range, which implies the existence of critical grading state at high stress level (Fig. 6).

Figure 4. Images of B023 under the normalized stress ($p/\sigma_0$) a) 0.00234, b) 0.416 and c) 2.34, respectively, and d) R023 under $p/\sigma_0 = 2.34$. 
Figure 5. a) Fractal grading and current grading (Wood and Maeda (11)). b) The grading change of B000 respect to the stress level. c) Compression curves of the rigid assemblies and the crushable assemblies. d) Change in the void ratio and the grading state index due to the stress level.

Figure 6. Mutual effect among the void ratio, e, normalized stress, \( p/\sigma_0 \), and grading state index, \( I_G \).
6. References


